FAR BEYOND

MAT122

Antiderivatives

Stony Brook University

Antiderivatives - Intro

If f' is a derivative of f then f is an <u>antiderivative</u> of f'

Likewise, f' is an antiderivative of f''

Notation: or integral $\int f(x)dx$

A function F is called an antiderivative of f on some interval I if $F'(x) = f(x)$ for all x on I.

$g(x) = \frac{1}{3}x^3$

$g'(x) = \frac{1}{3} \cdot 3x^2$

$g'(x) = x^2$ $\therefore$ if $f(x) = x^2$

then $F(x) = \frac{1}{3}x^3$ but …

$g(x) = \frac{1}{3}x^3 + 1$ F $\quad$ $g(x) = \frac{1}{3}x^3 + 7$ F $\quad$ $g(x) = \frac{1}{3}x^3 + C$

$g'(x) = x^2$ f $\quad$ $g'(x) = x^2$ f $\quad$ $g'(x) = x^2$

where C is a constant

Common Antiderivatives

If F is an antiderivative of f on I then the **general** antiderivative of f on I is $F(x)+C$ where C is an arbitrary constant.

To find the general antiderivative of a function, determine what it is the derivative of and add **+ C.**

given $f(x)=e^x$

then $F(x)=e^x+C$

given $f(x)=\dfrac{1}{x}$ derivative of $\ln x$

then $F(x)=\ln|x|+C$ domain of $\ln x$ is $(0,\infty)$

$|x|$ restricts domain from $\dfrac{1}{x}$

Antiderivative of Power Function

$$\text{if } f(x) = x^n$$

$$\text{then } F(x) = \frac{x^{n+1}}{n+1} + C$$

ex. $f(x) = x^2$

$$\text{then } F(x) = \frac{x^{2+1}}{2+1} + C$$

$$= \boxed{\frac{x^3}{3} + C}$$

ex. $f(x) = 8x^9 - 3x^6 + 12x^3$

$$\text{then } F(x) = \frac{8x^{10}}{10} - \frac{3x^7}{7} + \frac{12x^4}{4} + C$$

$$= \boxed{\frac{4}{5}x^{10} - \frac{3}{7}x^7 + 3x^4 + C}$$

ex. $f(x) = \frac{10}{x^9}$

$$= 10x^{-9}$$

$$F(x) = \frac{10x^{-8}}{-8} + C$$

$$= \boxed{-\frac{5}{4x^8} + C}$$

Antiderivative of Power Function - Do

Do: Find the general antiderivatives of the following:

if $f(x) = x^n$

then $F(x) = \dfrac{x^{n+1}}{n+1} + C$

$f(x) = 4x^3$

then $F(x) =$

$f(x) = 5x^9 - 14x^6 + 12x^3$

then $F(x) =$

ex. $f(x) = \dfrac{6}{x^2}$

then $F(x) =$